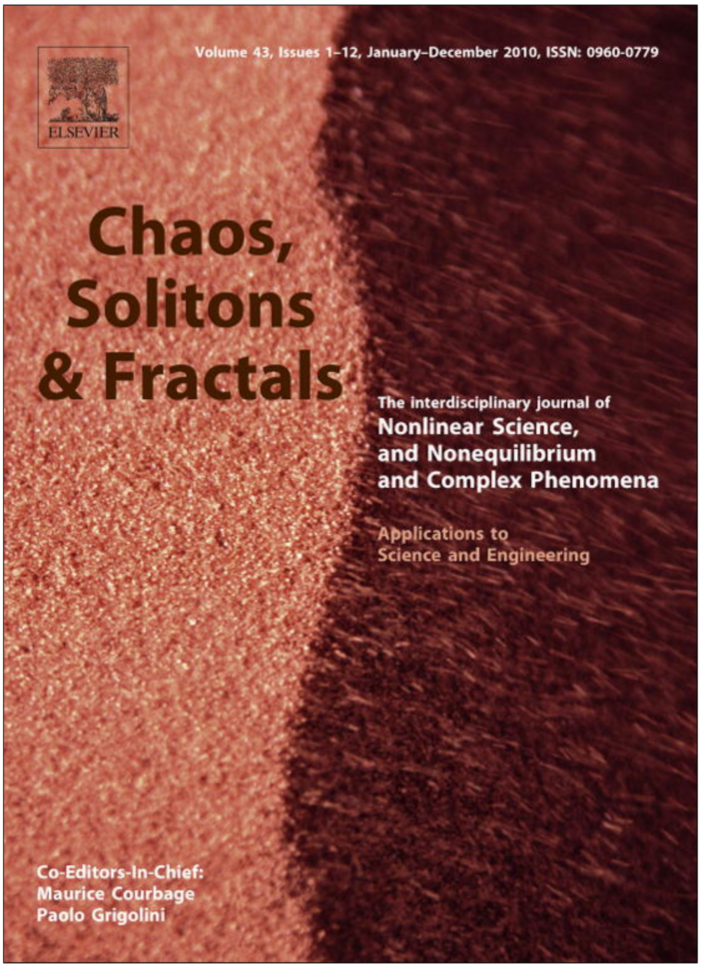
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On spurious anti-persistence in the US stock indices

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A R T I C L E   I N F O

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A B S T R A C T

We reexamine the results of Serletis and Rosenberg [Serletis A, Rosenberg A. Mean reversion in the US stock market. Chaos, Solitons and Fractals 2009;40:2007–2015.] who claim that the returns of the most important US stock indices (DJI, NASDAQ, NYSE and S&P500) are strongly anti-persistent and thus mean reverting. We apply various methods to detect long-range dependence – detrending moving average, detrended fluctuation analysis, generalized Hurst exponent approach, classical rescaled range analysis and modified rescaled range analysis. We show that there are no signs of anti-persistence in any of the indices. Moreover, we discuss that the authors did not find any anti-persistence but rather showed returns of the said assets do not follow the scaling power law around their moving average with varying window length. Anti-persistence is thus spurious and due to wrong application of detrending moving average method.

1. Introduction

Analysis of long-range dependence is important for financial analysis as its presence influences the basic implications for a risk management, a portfolio selection, an option pricing or trading strategies. Therefore, it is essential to efficiently and correctly detect such dependence in the time series.

We recall a self-similar process $X(t)$ for which it holds that $X(ct) \approx c^H X(t)$ in distribution for $t \geqslant 0$ and all $c > 0$. If the process $X(t)$ has stationary increments $X(t) - X(t-1)$, then the Hurst exponent H is a self-similarity parameter of process $X(t)$ which refers to the long-range dependence of the increments process [22]. The long-range dependence is present in a stationary time series when an autocorrelation function $\rho(k)$ of said process decays as $\rho(k) \approx Ck^{2H-2}$ for lag $k \to \infty$ where $0 < H < 1$ is Hurst exponent. The critical value of Hurst exponent is 0.5 and suggests two possible processes – either an independent or a short-term dependent process [15,4]. If $H > 0.5$, the auto-covariances of the process are significantly positive at all lags so that the process is persistent. On the other hand, if $H < 0.5$, the auto-covariances are significantly negative at all lags and the process is said to be anti-persistent. The persistent process implies that a positive movement is statistically more likely to be followed by another positive movement or vice versa. Reversely, the anti-persistent process implies that a positive movement is more statistically probable to be followed by a negative movement and *vice versa* [17]. In other words, the persistent process is trending whereas the anti-persistent process reverts more frequently than a random process.

This paper reacts to the results presented by Serletis and Rosenberg [23] who claim to find a significant negative long-range dependence (anti-persistence) in the returns of the most important indices of the US stock markets – NASDAQ Composite Index (NASDAQ), Dow Jones Industrial Average (DJI), S&P500 index (SPX) and NYSE Composite Index (NYSE). We reexamine the exact same period of the time series and show that such a strong anti-persistence is spurious and it is found due to an incorrect use of detrending moving average method [6]. Robustness of our results is checked by other frequently used methods – detrended fluctuation analysis (DFA) [20,13], generalized Hurst exponent approach (GHE) [9], rescaled range analysis (R/S) [11] and modified rescaled range analysis (M-R/S) [16].

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In their paper, [23] claim that the returns of the US indices between 5th February 1971 and 1st December 2006 can be characterized by the Hurst exponent of 0.0631, 0.0636, 0.0692 and 0.0672 for DJI, SPX, NASDAQ and NYSE, respectively. Such results indicate strong anti-persistence and it would imply that there was a significant potential for above-average profit strategies as the direction of the returns would be strongly predictable in such case (the opposite sign of the upcoming return with respect to the current one would be almost certain). We analyze the same dataset and show that such anti-persistence is spurious.

The paper is structured as follows. In Section 2, we focus on the mostly used methods of the Hurst exponent estimation – DMA, DFA, GHE, R/S and M-R/S. In Section 3, we present a basic description of the data set. In Section 4, we show the results of the long-range dependence detection. In Section 5, the spurious anti-persistence is discussed. Section 6 concludes. We show that there are no signs of significant anti-persistence in any of the examined indices. We also discuss that the spurious anti-persistence found by Serletis and Rosenberg [23] is rather a sign of the fact that the deviations of returns from their moving average do not follow scaling power law but rather stabilize for longer windows of the moving averages.

2. Hurst exponent estimation methods

In this section, we present very brief description of DMA, DFA, GHE, R/S and M-R/S. For more details, see the mentioned references.

2.1. Detrending moving average

Detrending moving average (DMA), proposed by Alessio et al. [1], is based on a filtering of the original series by moving averages with different window sizes. For the time series of length T, the moving average with the window size of λ for each point $X(t)$ is defined as $\overline{X}_\lambda(t) = \sum_{k=0}^{\lambda-1} X(t-k)/\lambda$. Fluctuations $F^2_{DMA}(\lambda)$, defined as the mean squared error of $X(t)$ from $\overline{X}_\lambda(t)$, scale as $F^2_{DMA}(\lambda) \approx c\lambda^{2H}$.

DMA is sensitive to a choice of λ. For our research, we use $\lambda_{min} = 2$ and $\lambda_{max} = 50$ so that H is estimated from 49 points to get comparable results with the discussed paper of [23]. The case of $\lambda = 1$ is omitted as it implies $F^2_{DMA}(1) = 0$ for any series and thus gives no information about the studied process.

2.2. Detrended fluctuation analysis

Detrended fluctuation analysis (*DFA*) was proposed by Peng et al. [20] while examining series of DNA nucleotides. In the procedure, the time series of length T is divided into sub-periods of length υ and the profile (cumulative deviations from the mean) is constructed. The linear fit $X_\upsilon(t)$ of the profile is estimated for each sub-period. A detrended signal $Y_\upsilon(t)$ is then constructed as $Y_\upsilon(t) = X(t) - X_\upsilon(t)$. Fluctuation $F^2_{DFA}(\upsilon)$, defined as an average mean squared error from the linear fit over all sub-periods of length υ, scales as $F^2_{DFA}(\upsilon) \approx c\upsilon^{2H}$, where c is a constant independent of υ [13].

As DFA is based on linear fitting and averaging over sub-periods, minimum sub-period length υ_{min} as well as maximum length υ_{max} needs to be set to avoid inefficient fitting and averaging. In the research, we use $\upsilon_{min} = 10$ and $\upsilon_{max} = T/5$ proposed in several studies [10,2,19].

2.3. Generalized Hurst exponent

Generalized Hurst exponent approach (GHE), proposed for financial time series by Di Matteo et al. [9], is based on scaling of qth order moments of the distribution of the increments of the process $X(t)$. The scaling is characterized on the basis of the statistic $K_q(\tau)$ defined as $K_q(\tau) = \sum_{t=0}^{T-\tau}(|X(t+\tau) - X(t)|^q)/(T-\tau)$ for time series of length T. Parameter τ can be understood as an "investment horizon" in financial terms. The statistic scales as $K_q(\tau) \approx c\tau^{qH(q)}$. For the purposes of the long-range dependence detection, we set $q = 2$ so that Hurst exponent $H(2)$ is estimated from relationship $K_2(\tau) \approx c\tau^{2H(2)}$.

To get comparable results with DMA, we set $\tau_{min} = 1$ and $\tau_{max} = 50$ so that $H(2)$ is estimated from 50 points. Such choice of parameters takes investment horizons from 1 to 50 trading days into consideration.

Note that for all the presented methods, the process $X(t)$ is the time series of logarithmic prices of the examined assets to evaluate the potential long-range dependence in the logarithmic (continuous) returns, i.e. we estimate the Hurst exponent H as the self-similarity parameter for the process of logarithmic prices of the indices which also characterizes the long-range dependence in the process of increments of the original process, i.e. the logarithmic returns.

2.4. Classical and modified rescaled range analyses

Rescaled range analysis (R/S) can be used as either a parametric (estimating H) or non-parametric method (testing the presence of long-range dependence) [11,18]. As the method was shown to be biased even for independent processes [14,16], we use the non-parametric version of the method to test the presence of long-range dependence solely. For the increments time series of length T, we construct a rescaled range of the series $(R/S)_T$ as a fraction of a range of a profile (cumulative deviations from an arithmetic mean) and a standard deviation of the increments process.

V statistic is defined as

$$V_T = \frac{(R/S)_T}{\sqrt{T}}$$

and converges to a distribution defined as $F_V(x) = 1 + 2\sum_{k=1}^{\infty}(1 - 4k^2x^2)e^{-2(kx)^2}$ for independent processes [16,11,21]. As R/S analysis presented above (usually called classical) is biased by a presence of short-term memory, [16] proposed modified rescaled range analysis ($M - R/S$) which differs from the classical one by a use of a modified standard deviation S^M, which is defined with a use of autocovariances of the increments series γ_j up to lag ξ as

$$S_{I_n}^M = \sqrt{S_{I_n}^2 + 2\sum_{j=1}^{\xi}\gamma_j\left(1 - \frac{j}{\xi+1}\right)}. \quad (1)$$

The distribution of the modified V statistic converges to F_V not only for independent processes but also for short-range dependent ones.

3. Data and initial analysis

We reexamine the same sample period as in [23] – DJI, NASDAQ, NYSE, SPX between February 5th 1971 (foundation of NASDAQ Composite index) and December 1st 2006. Basic descriptive statistics are summarized in Table 1 and Fig. 1 shows the logarithmic returns for the examined period. The evolution of the indices is very similar while the biggest differences are present during the market crash of 1978 – NASDAQ was hit in lower magnitude when compared to the other indices – and after the "DotCom" bubble burst – NASDAQ shows the highest increase in volatility compared to the other examined indices.

The returns of all examined indices posses typical characteristics of financial time series – negative skewness and excess kurtosis [7]. Normality of the returns is strongly rejected by both Jarque–Bera and Shapiro–Wilk tests. First-order autocorrelation of the returns are positive for all four indices – 0.05, 0.10, 0.10, 0.06 for DJI, NASDAQ, NYSE and SPX, respectively. A non-presence of significant autocorrelations in the first 20 lags is strongly rejected by standard Box-Pierce Q-test for all the indices. Finally, all indices are stationary as KPSS test does not reject stationarity for any index while ADF test rejects a unit-root for all indices. For the test statistics and p-values, see Table 1.

We further illustrate the evolution of the autocorrelations by autocorrelation functions (ACF) and partial autocorrelation functions (PACF) in Fig. 2. For each index, there are several significant autocorrelations at different lags. A simple examination of ACF and PACF of the indices can tell important information about potential presence of anti-persistence. In Fig. 3, ACF and PACF of two popular long-range dependence models – fractional Gaussian noise (fGn) and autoregressive fractionally integrated moving average (ARFIMA) – with $H = 0.1$ (strong anti-persistence close to the one claimed to be found in [23]) and $T = 9040$ are presented. When the autocorrelation functions of the US indices are compared with the ones of the artificial processes, the significant differences are obvious. Firstly, very significant negative autocorrelation is present at the first lag for both fGn and ARFIMA. Secondly, PACF is significantly negative at all 20 lags while the autocorrelations are slowly increasing (getting closer to zero) with the increasing lag. None of these are present in ACF and PACF of DJI, NASDAQ, NYSE and SPX. Such a strong anti-persistence of the returns as argued in [23] can be rejected by the simple examination of ACF and PACF. Nevertheless, we follow with more rigorous and deeper analysis of potential long-range dependence in the returns of the examined US stock indices.

4. Results

In the previous section, we have shown that all of the examined indices share the same basic statistical properties – negative skewness, leptokurtosis, non-normality, positive first-order autocorrelations, fast decay of autocorrelations with increasing lag k and stationarity. Thus there are no clear signs of long-range dependence visible from these basic properties. Detailed examination of potential long-range dependence follows.

We start with the parametric methods (i.e. estimating Hurst exponent) – DMA, GHE, and DFA. The estimated Hurst exponents and corresponding coefficient standard errors for all four indices are presented in Table 2. Hurst exponents based on GHE and DFA show quite homogeneous results – there is no long-range dependence in DJI, NYSE and SPX as the estimates of Hurst exponents cannot be statistically distinguished from $H = 0.5$ while NASDAQ shows weak signs of persistence (at 99% level of significance). In general, NASDAQ is characterized by the highest

Table 1
Descriptive statistics.

	DJI	NASDAQ	NYSE	S&P500
Mean	0.00029	0.00035	0.00031	0.00030
SD	0.01021	0.01189	0.00916	0.00993
Max	0.09666	0.13255	0.08622	0.08709
Min	−0.25632	−0.12043	−0.21286	−0.22900
Skewness	−1.80491	−0.30851	−1.53977	−1.43878
Excess kurtosis	48.25733	10.75666	36.79083	35.54557
Observations	9043	9040	9045	9043
Jarque–Bera	882,371	43,726	513,699	479,192
p-Value	0.0000	0.0000	0.0000	0.0000
Shapiro–Wilk	0.9101	0.8866	0.9171	0.9188
p-Value	0.0000	0.0000	0.0000	0.0000
$\rho(1)$	0.0478	0.1009	0.1037	0.0590
$Q(20)$	55.1648	191.6704	127.2067	69.7841
p-Value	0.0000	0.0000	0.0000	0.0000
KPSS	0.1813	0.0874	0.1327	0.1349
5% critical value	0.4630	0.4630	0.4630	0.4630
ADF	−26.7642	−23.1461	−26.4745	−26.7512
p-Value	0.0000	0.0000	0.0000	0.0000

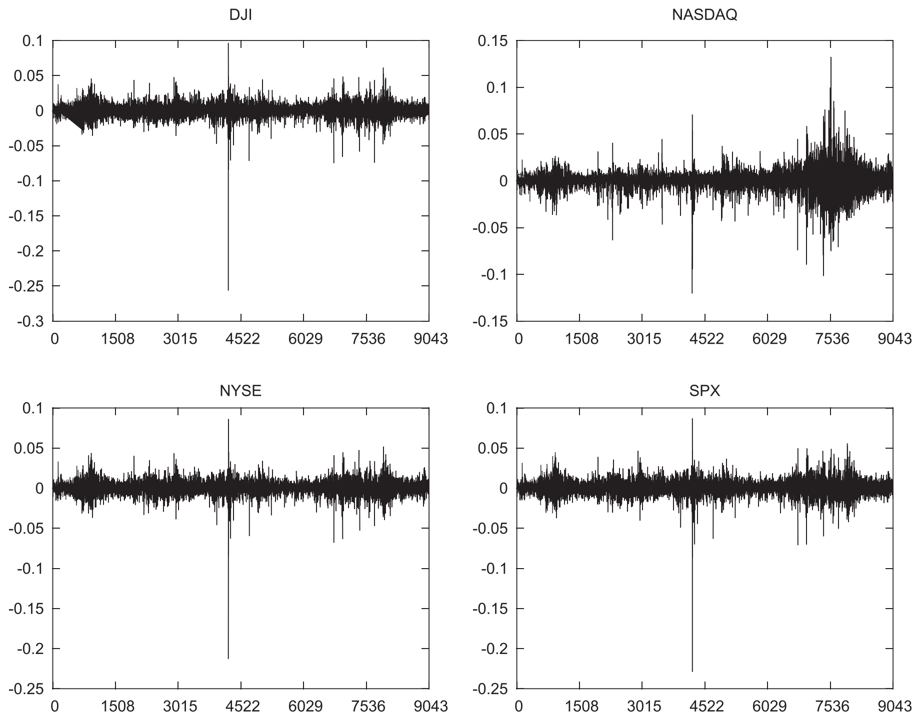

Fig. 1. Logarithmic returns of the US indices.

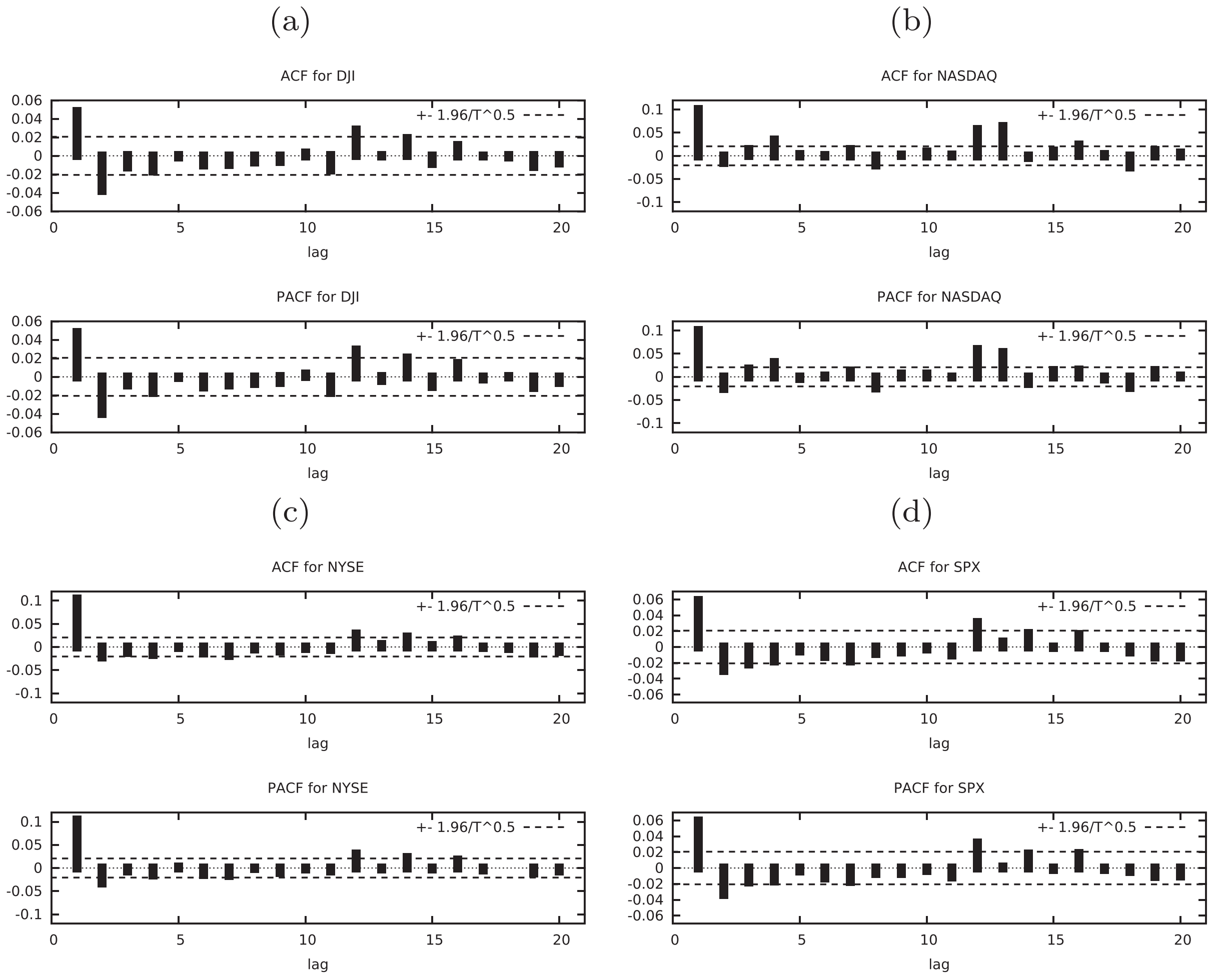

Fig. 2. ACF and PACF of DJI, NASDAQ, NYSE and S&P500 in (a)–(d), respectively.

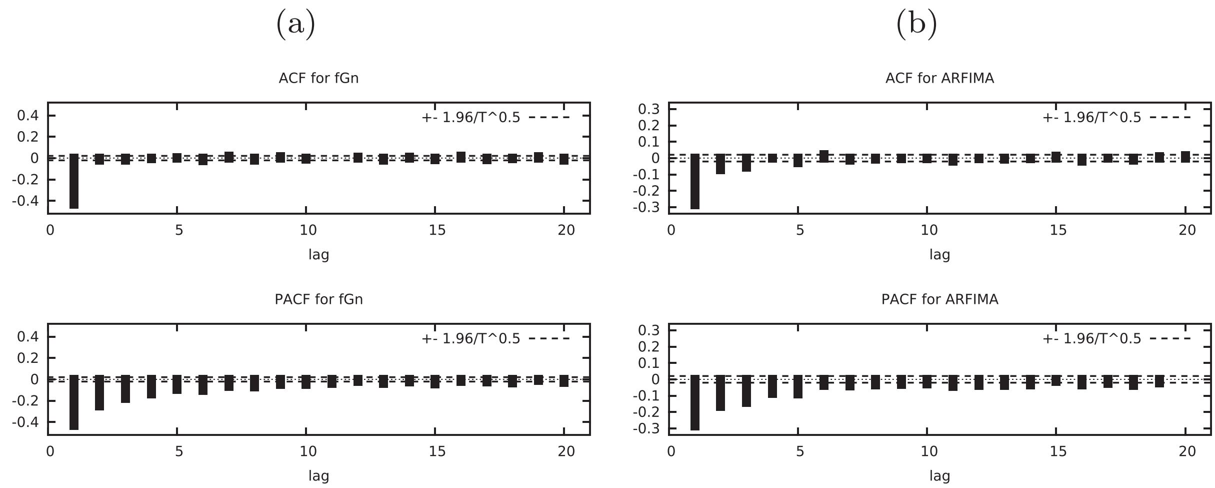

Fig. 3. ACF and PACF of fGn and ARFIMA with $H = 0.1$ in (a) and (b), respectively.

Table 2
Hurst exponent estimates and standard errors.

	DMA	GHE	DFA
DJI	0.5766 (0.0222)	0.4877 (0.0044)	0.4645 (0.0009)
NASDAQ	0.6544 (0.0196)	0.5639 (0.0064)	0.5533 (0.0012)
NYSE	0.5957 (0.0289)	0.5057 (0.0072)	0.4887 (0.0006)
SPX	0.5769 (0.0227)	0.4891 (0.0053)	0.4995 (0.0009)

Hurst exponents for all three methods when compared to DJI, NYSE and SPX. DMA method yields higher estimates than the other methods by approximately 0.1 for all the indices. To further examine the reason for this difference, we analyze the scaling of $F^2_{DMA}(\lambda)$, $F^2_{DFA}(\upsilon)$ and $K_2(\tau)$.

Figs. 4, 6 and 8 show the scaling of $F^2_{DMA}(\lambda)$, $F^2_{DFA}(\upsilon)$ and $K_2(\tau)$ for DMA, GHE and DFA, respectively. For DMA, $F^2_{DMA}(\lambda)$ increases concavely for low λs and scales linearly for higher scales for all four indices. Such behavior indicates that the estimates are affected by the presence of short-range dependence in the process, which has actually been shown to be present in all the examined processes in the previous section. For GHE, $K_2(\tau)$ behaves differently and shows potential crossover between two linear scaling laws approximately in the middle of the scaling range. However, the crossover is quite weak for all the examined indices. Nevertheless, the crossover can be existent again due to the presence of short-range dependence in the processes. For DFA, $F^2_{DFA}(\upsilon)$ shows quite stable linear scaling for majority of scales υ. However, for higher scales, the scaling becomes unstable and volatile as $F^2_{DFA}(\upsilon)$ is based on low number of averaged values.

As the scaling obviously changes with different scales, we observe the multi-scaling phenomena which in this case is most probably caused by the presence of short-range dependence in the examined processes. To examine the multi-scaling in more detail, we focus on the estimates of Hurst exponents when a different number of points is

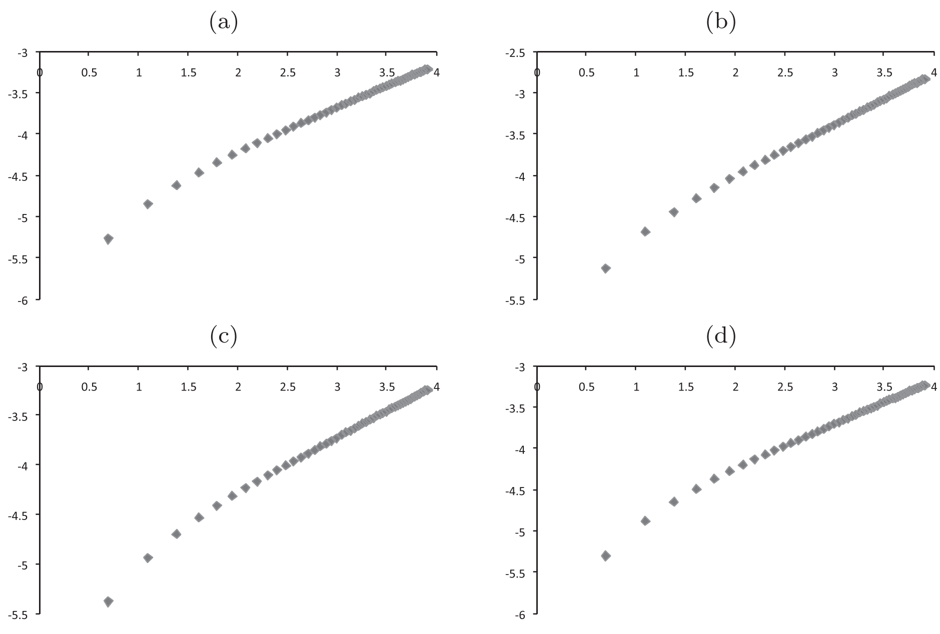

Fig. 4. Scaling of $F^2_{DMA}(\lambda)$ on y-axis with changing λ on x-axis for $\lambda = 2, 3, \ldots, 49, 50$ for DJI, NASDAQ, NYSE and SPX for (a)–(d), respectively.

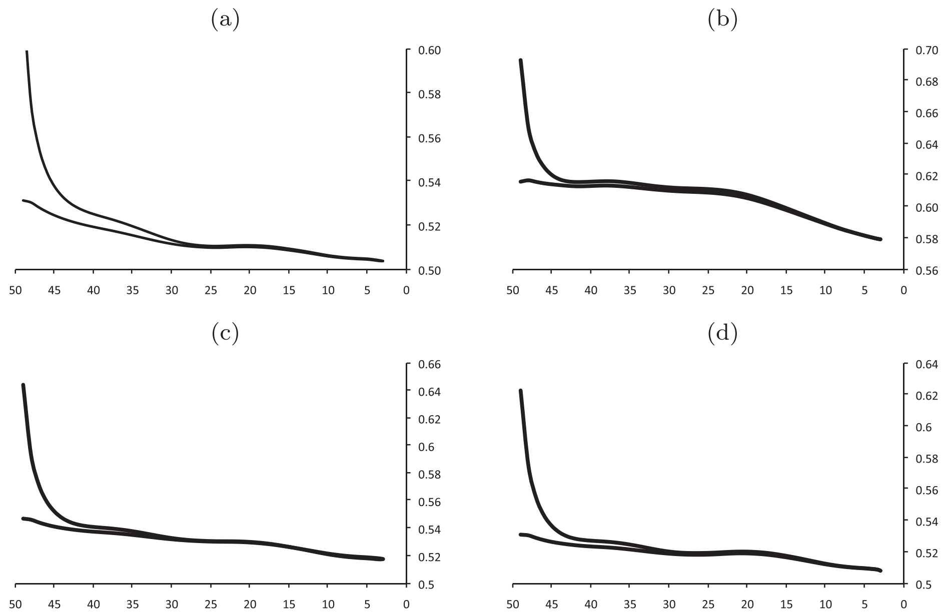

Fig. 5. 95% confidence intervals for the estimates of Hurst exponent based on DMA (y-axis) with changing number of λs taken into consideration (x-axis) for DJI, NASDAQ, NYSE and SPX for (a)–(d), respectively. For 50 points taken into consideration, all λs are used for the estimation; for three points, only λ = 48, 49, 50 are used for the estimation of Hurst exponent.

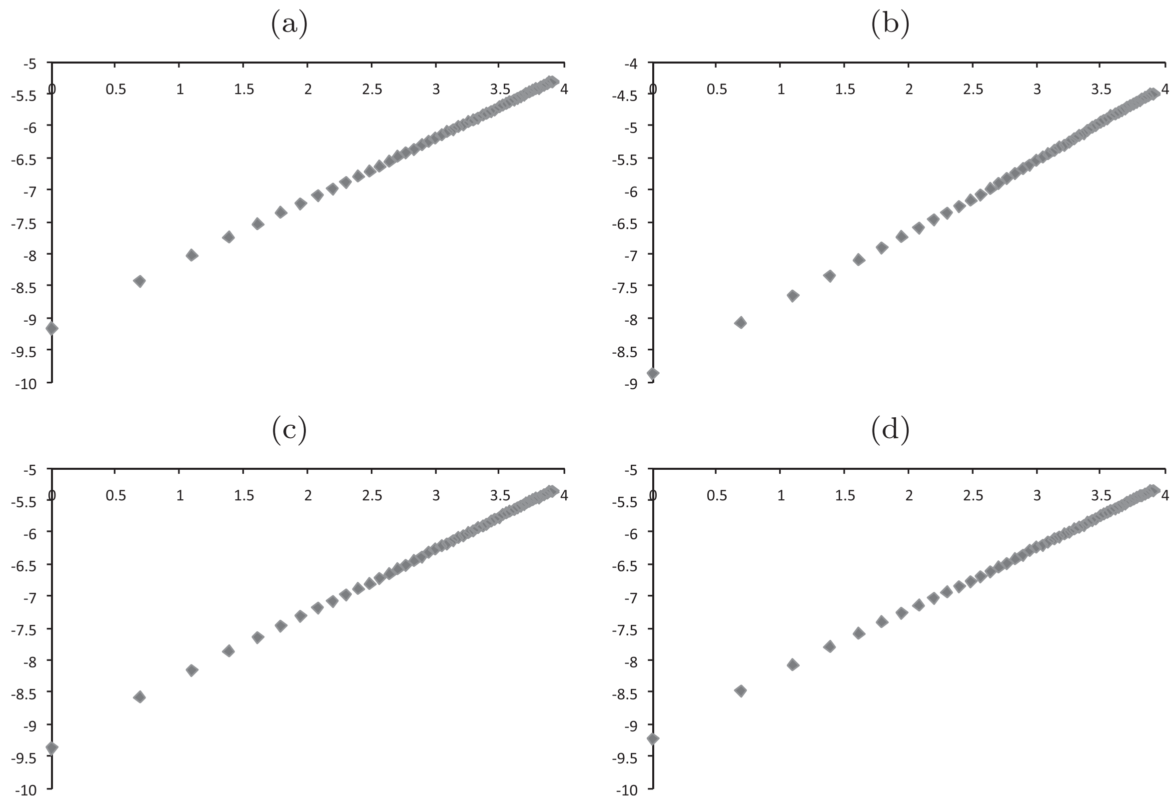

Fig. 6. Scaling of $K_2(\tau)$ on y-axis with changing τ on x-axis for $\tau = 1, 2, \ldots, 49, 50$ for DJI, NASDAQ, NYSE and SPX for (a)–(d), respectively.

taken into consideration for the final regression. Figs. 5, 7 and 9 show the estimates for DJI, NASDAQ, NYSE and SPX based on DMA, GHE and DFA, respectively. In the Figures, we show the estimates of Hurst exponents for varying number of scales taken into consideration. As the long-range dependence is defined as an asymptotic property, we start with a full range of scales and decrease the range in steps to eventually estimate Hurst exponents on the three highest scales for either $F^2_{DMA}(\lambda)$ or $K_2(\tau)$ or $F^2_{DFA}(\upsilon)$. The results support our findings and for DMA and GHE, even the estimates of Hurst exponents based on low number of statistics are stable.

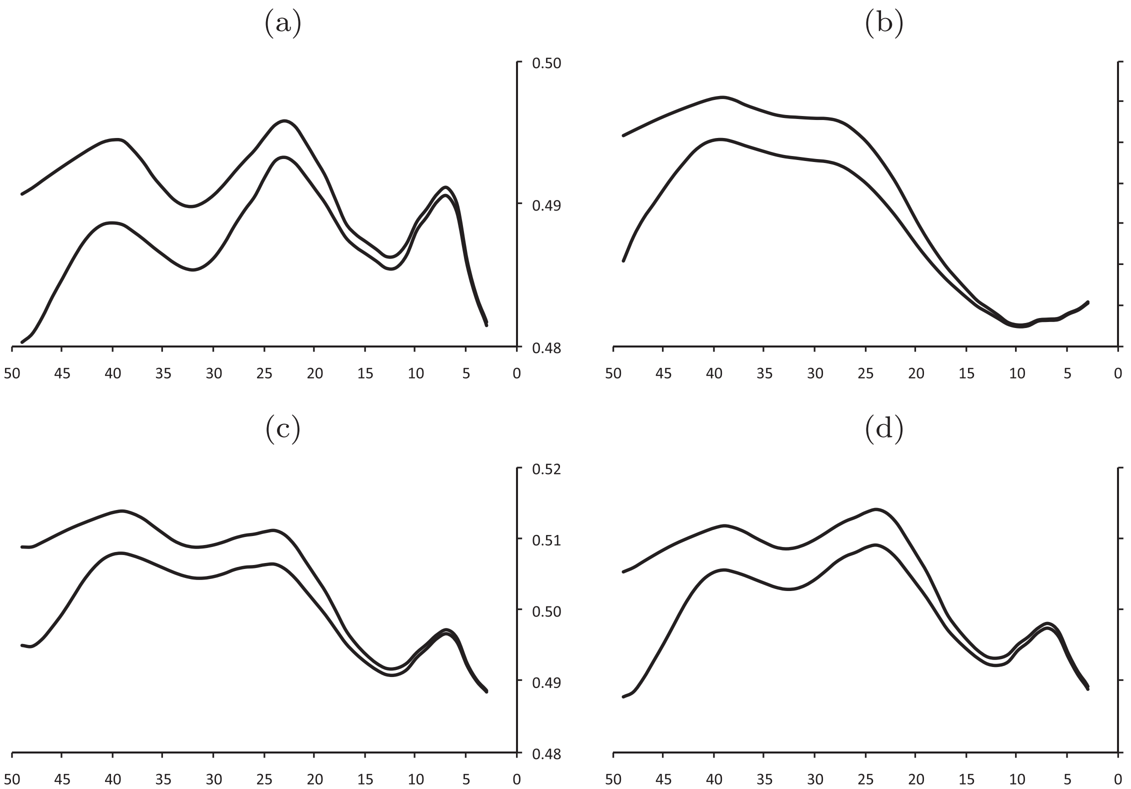

Fig. 7. 95% confidence intervals for the estimates of Hurst exponent based on GHE (y-axis) with changing number of τs taken into consideration (x-axis) for DJI, NASDAQ, NYSE and SPX for (a)–(d), respectively. For 50 points taken into consideration, all τs are used for the estimation; for three points, only τ = 48, 49, 50 are used for the estimation of Hurst exponent.

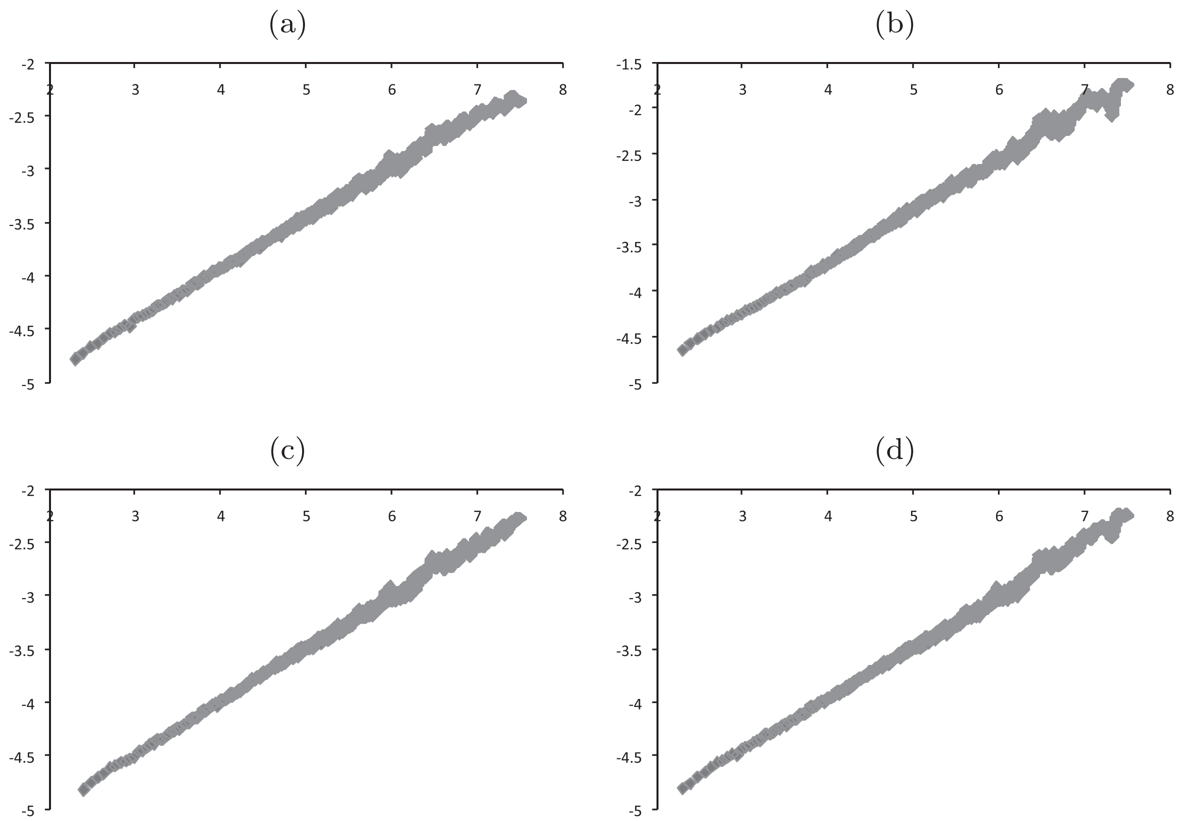

Fig. 8. Scaling of $F^2_{DFA}(\upsilon)$ on y-axis with changing υ on x-axis for $\upsilon = 5, 6, \ldots, \frac{T}{5} - 1, \frac{T}{5}$ for DJI, NASDAQ, NYSE and SPX for (a)–(d), respectively.

For DMA, the estimates of Hurst exponents slowly converge to support the earlier findings – the estimates for DJI, NYSE and SPX converge to the values close to 0.5 while the estimate of NASDAQ converge to 0.58. The estimates based on GHE are more stable and show the same results as DMA – NASDAQ is weakly persistent while there are no signs of long-range dependence for the other indices. The situation is most complicated for DFA. Even though the scaling of $F^2_{DFA}(\upsilon)$ seems linear, it is rather unstable when we look at the estimates of Hurst exponents based on different scales υ. The evolution of the estimates indicates that $\upsilon_{max} = T/5$ is too restrictive and averaging of $F^2_{DFA}(\upsilon)$ over only five of these statistics yields inefficient and unstable results, which biases the final estimate. This fact makes the results of DFA hardly usable apart from the most basic estimates of Hurst exponent taking all scales into consideration.

We follow with the non-parametric methods (i.e. testing presence of long-range dependence in the process) – R/S and M-R/S. As most of the Hurst exponent estimators are biased by the presence of short-range dependence in the process [16,12] as well as various distributional properties [3], we use two types of moving block bootstrapping – a simple one and a one with pre-whitening and post-blackening [24] – as well as simple shuffling for a construction of the confidence intervals for hypothesis testing. For moving block bootstrap method, the corresponding V statistic is constructed for the new series made of shuffled blocks of 10 observations from the original series. For pre-whitening and post-blackening, we apply AR(1) process. For each series, bootstrapping and shuffling is repeated 1000 times. With a use of shuffling, we obtain confidence intervals which are robust to different distributional properties. Moreover, with a use of moving block bootstrap, we obtain confidence intervals which are robust to short-range dependence, heteroskedasticity and various distributional properties of the original series. Therefore, we can distinguish between the effects of distribution, short-range dependence and potential presence of long-range dependence in the examined series.

In Tables 3–5, we present the estimates of V statistics for the stock indices. We have estimated the statistics for the lags $\xi = 0, 1, \ldots, 10$ and in the same way, the confidence intervals (two-tailed at 5% significance level) have been constructed. From Table 3, we can say that independence cannot be rejected for three indices – DJI, NYSE and SPX – while for NASDAQ, we reject independence for 7 out of 11 different lags taken into consideration. For the bootstrapped confidence intervals, there is no single value of V statistic falling outside the intervals indicating no long-range dependence. Therefore, the results based on moving block bootstrap support the findings about DJI, NYSE and SPX and specify the type of dependence in NASDAQ as a short-range one. Thus, there are no signs of significant long-range dependence in the returns processes of all the examined indices. These results again contradict the ones of [23] who claimed that all the returns of the indices are strongly negatively long-range dependent.

Overall, all used methods – R/S, M-R/S, GHE, DMA and DFA – indicate that none of the indices can be characterized as the anti-persistent process. Even though the estimates of Hurst exponent for NASDAQ indicate weak

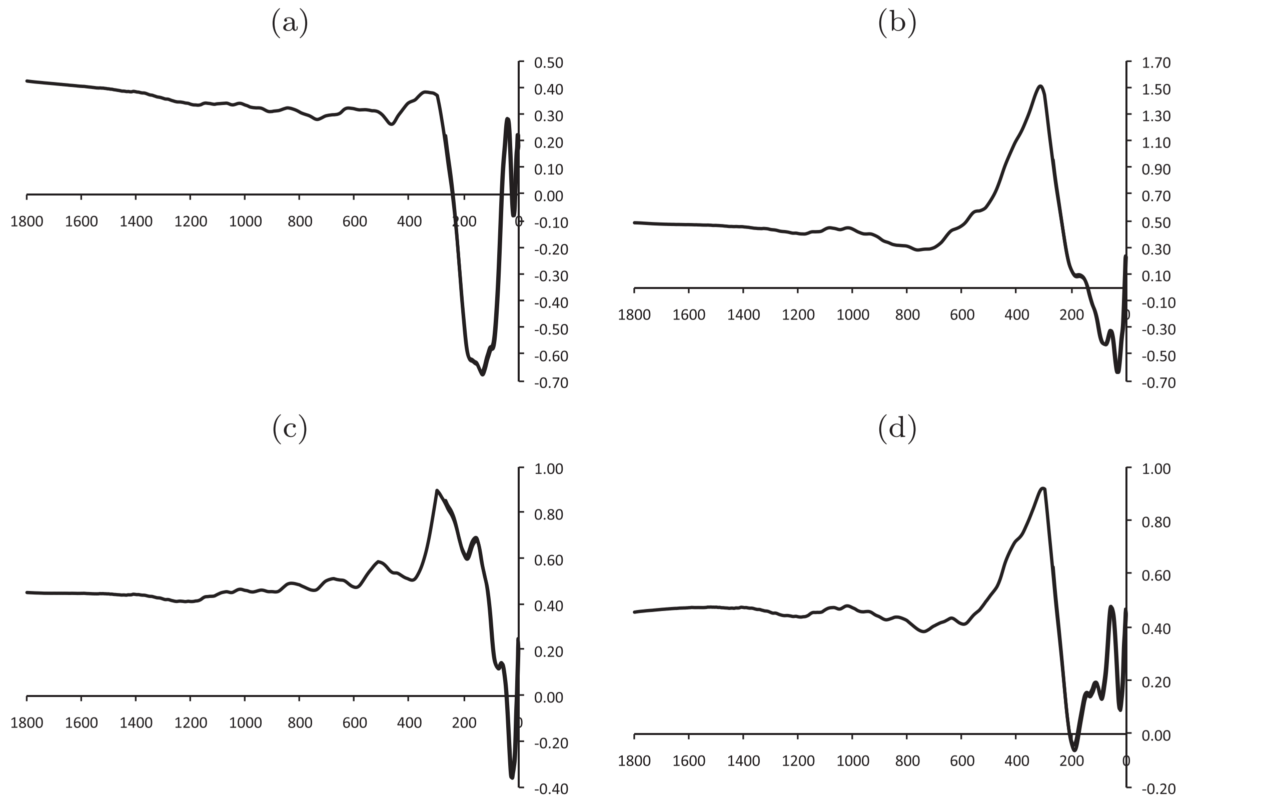

Fig. 9. 95% confidence intervals for the estimates of Hurst exponent based on DFA (y-axis) with changing number of υs taken into consideration (x-axis) for DJI, NASDAQ, NYSE and SPX for (a)–(d), respectively. The rest of the notation holds accordingly.

Table 3
V statistics for shuffled time series.

Lag		V statistic	$P_{2.5}$	$P_{97.5}$		V statistic	$P_{2.5}$	$P_{97.5}$
0	DJI	1.4760	0.7979	1.8412	NASDAQ	*1.9986**	0.7855	1.7867
1		1.4419	0.7870	1.8884		*1.9048**	0.7868	1.8563
2		1.4481	0.7797	1.8020		*1.8841**	0.8207	1.8440
3		1.4555	0.7817	1.8495		*1.8685**	0.8069	1.8343
4		1.4646	0.7899	1.7973		*1.8483**	0.7860	1.8433
5		1.4712	0.8084	1.8697		1.8344	0.7715	1.8887
6		1.4779	0.7754	1.7942		*1.8245**	0.7846	1.8021
7		1.4848	0.8081	1.8497		1.8145	0.8005	1.8489
8		1.4913	0.7838	1.7857		1.8101	0.8073	1.8232
9		1.4974	0.7895	1.8488		1.8064	0.8005	1.8310
10		1.5020	0.7999	1.8168		*1.8022**	0.7747	1.7845
0	NYSE	1.2875	0.8185	1.8162	SPX	1.4731	0.7905	1.8727
1		1.2257	0.7897	1.8899		1.4316	0.7851	1.8628
2		1.2146	0.8130	1.8508		1.4318	0.8064	1.8222
3		1.2124	0.8237	1.8506		1.4390	0.8026	1.8634
4		1.2145	0.8026	1.8131		1.4483	0.7685	1.8559
5		1.2160	0.8123	1.8019		1.4557	0.8132	1.7938
6		1.2192	0.7905	1.8189		1.4634	0.7951	1.8456
7		1.2241	0.7867	1.8025		1.4726	0.8101	1.8505
8		1.2285	0.7911	1.7938		1.4812	0.7792	1.8490
9		1.2330	0.8039	1.8814		1.4891	0.8307	1.8175
10		1.2369	0.7976	1.8197		1.4959	0.7611	1.8426

5% significance is marked with asterisk * and italics.

Table 4
V statistics for moving block bootstrapping with pre-whitening and post-blackening.

Lag		V statistic	$P_{2.5}$	$P_{97.5}$		V statistic	$P_{2.5}$	$P_{97.5}$
0	DJI	1.4760	0.8244	1.7627	NASDAQ	1.9986	0.9017	2.0822
1		1.4419	0.7807	1.8007		1.9048	0.8386	1.9437
2		1.4481	0.8025	1.7823		1.8841	0.8585	1.9817
3		1.4555	0.7774	1.7649		1.8685	0.8541	1.9247
4		1.4646	0.7666	1.7784		1.8483	0.8356	1.9454
5		1.4712	0.7686	1.8196		1.8344	0.8083	1.9377
6		1.4779	0.7736	1.8592		1.8245	0.8358	1.8907
7		1.4848	0.7702	1.7975		1.8145	0.8190	1.8925
8		1.4913	0.8119	1.8291		1.8101	0.8113	1.8713
9		1.4974	0.7896	1.7777		1.8064	0.8066	1.8755
10		1.5020	0.7942	1.7647		1.8022	0.8046	1.8528
0	NYSE	1.2875	0.8228	1.9798	SPX	1.4731	0.7776	1.8410
1		1.2257	0.8209	1.8914		1.4316	0.8059	1.7727
2		1.2146	0.7878	1.8045		1.4318	0.7781	1.7931
3		1.2124	0.7943	1.8224		1.4390	0.7908	1.7541
4		1.2145	0.7734	1.8557		1.4483	0.7809	1.6836
5		1.2160	0.7962	1.8398		1.4557	0.7857	1.8550
6		1.2192	0.8049	1.8401		1.4634	0.7832	1.8568
7		1.2241	0.8034	1.8466		1.4726	0.7889	1.7767
8		1.2285	0.8013	1.7768		1.4812	0.8250	1.7836
9		1.2330	0.7769	1.8341		1.4891	0.7801	1.8290
10		1.2369	0.8089	1.8135		1.4959	0.7707	1.8318

persistence, we have shown that it is rather an effect of short-range dependence on the estimators than a true persistence. Therefore, neither persistence characterizes any of the indices. Altogether, none of the examined indices can be characterized by long-range dependence of any type.

5. Discussion

As our results are very different from the ones of [23], it raises an important question about the reason why it is so. The results we present are robust as the outcomes are almost equal for all the used methods – DMA, DFA, GHE, R/S and M-R/S. Moreover, the results are in hand with the findings of other authors showing only weak or no long-range dependence in financial returns at all [8,5].

The problem seems to be in the application of DMA itself. The authors most likely set $X(t) = r(t)$ for detection of long-range dependence in returns process $r(t) = \log P(t) - \log P(t-1)$. However, the examination of deviations of prices around their moving average by the means of DMA is actually examination of self-similarity of the process and potential long-range dependence in the increments of the process [22]. Therefore, to examine the

Table 5
V statistics for simple moving block bootstrapping.

Lag		V statistic	$P_{2.5}$	$P_{97.5}$		V statistic	$P_{2.5}$	$P_{97.5}$
0	DJI	1.4760	0.7902	1.8323	NASDAQ	1.9986	0.9165	2.0971
1		1.4419	0.7878	1.7383		1.9048	0.8345	1.9424
2		1.4481	0.7883	1.8187		1.8841	0.8689	1.9483
3		1.4555	0.7538	1.7595		1.8685	0.8435	1.9274
4		1.4646	0.8066	1.8106		1.8483	0.8072	1.9567
5		1.4712	0.7846	1.7712		1.8344	0.8096	1.9008
6		1.4779	0.7845	1.8442		1.8245	0.8180	1.8489
7		1.4848	0.7549	1.8011		1.8145	0.8055	1.8617
8		1.4913	0.7676	1.7728		1.8101	0.8052	1.8733
9		1.4974	0.7891	1.8013		1.8064	0.8256	1.8660
10		1.5020	0.8023	1.8104		1.8022	0.7934	1.8553
0	NYSE	1.2875	0.8352	1.9541	NASDAQ	1.4731	0.8040	1.8495
1		1.2257	0.8007	1.8608		1.4316	0.7822	1.7648
2		1.2146	0.7900	1.8303		1.4318	0.7810	1.7217
3		1.2124	0.7643	1.7935		1.4390	0.7985	1.7701
4		1.2145	0.7990	1.7703		1.4483	0.7869	1.7359
5		1.2160	0.7824	1.8027		1.4557	0.7899	1.7509
6		1.2192	0.7927	1.7867		1.4634	0.7886	1.8101
7		1.2241	0.8062	1.7990		1.4726	0.7914	1.7584
8		1.2285	0.8037	1.8414		1.4812	0.7799	1.8257
9		1.2330	0.7854	1.8425		1.4891	0.8072	1.8335
10		1.2369	0.8151	1.7979		1.4959	0.8002	1.8233

long-range dependence in the logarithmic returns $r(t)$ of the process, we need to set $X(t) = \log P(t)$ where $P(t)$ is an index price at time t.[1]

This turns the detection of long-range dependence in returns in [23] into examination of deviations of returns from their moving average with increasing window length λ. However, the average of the financial returns stabilizes, after some variation for short windows, around a constant value (usually close to 0) so that the mean squared deviations do not change or change only slightly with the increasing window. This might be wrongly interpreted as strong anti-persistence but is only the reflection of the fact that returns do not scale (at all or only slightly) around their moving average according to any power law. A fact that the scaling of $F^2_{DMA}(\lambda)$ vanishes for higher values of λ is visible in the Figures presented in [23].

Alternatively, the anti-persistence might be correctly detected but for the second differences of the initial process and not the returns (the first differences) themselves. This would indicate that the process $r_{2nd}(t)$ defined as $r_{2nd}(t) = \log P(t) - 2\log P(t-1) + \log P(t-2)$ for $t = 2, 3, \ldots, T$ is strongly anti-persistent. In financial terms, if the return of today has increased compared to the return of yesterday, it will most likely decrease tomorrow compared to the return of today. However, this only means that the process of returns is not explosive and it has no additional informative value. Moreover, detrending methods might be biased for processes with true Hurst exponent very close to zero which was already discussed for DFA in [13] but it has not been for DMA yet. Thus statistical inference based on DMA might be biased for very low H. Again, Figures in [23] which illustrate the scaling of $F^2_{DMA}(\lambda)$ show that apart from low scales, the scaling is practically non-present and Hurst exponents would be estimated even closer to zero if only the higher scales were taken into consideration.

6. Conclusions

We reexamined the results of [23] who claimed to find strong anti-persistence in the US stock indices – DJI, NASDAQ, NYSE and SPX – with the use of detrending moving average, detrended fluctuation analysis, generalized Hurst exponent approach, classical rescaled range analysis and modified rescaled range analysis. We show that there are no signs of anti-persistence (or long-range dependence in general) in the processes of returns of the examined indices. We also discuss that the spurious anti-persistence is due to a misinterpretation and is only a sign of incorrect implementation of detrending moving average method.

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[1] To support our assertion of an incorrect use of the method, we estimated Hurst exponents with DMA, setting $X(t) = r(t) = \log P(t) - \log P(t-1)$, and arrived at estimates very close to the ones of [23], i.e. in the interval $H \in (0.06; 0.07)$. The fact that the estimates are not exactly equal is most likely caused by non-identical data sets as we obtained the data from `finance.yahoo.com` while a source of data is not given in [23]. Nevertheless, such results support our claim that anti-persistence in the US stock indices was detected by Serletis and Rosenberg [23] due to an incorrect application of DMA method.

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